Hypernucleus Production by $A(p, pK^+)_\Lambda B$ Reactions

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Abstract

The Λ-hypernucleus production by $A(p, pK^+)_\Lambda B$ reactions is investigated within the framework of the distorted wave impulse approximation(DWIA). The amplitude for the elementary process is evaluated in a fully covariant two-nucleon model based on the effective Lagrangian. The reaction cross sections for Λ-hypernucleus productions on ^6Li, ^{12}C and ^{16}O targets are calculated. It is found that the distortion effects tend to reduce the cross sections by a factor of 3∼10. Various differential cross sections (DCS) and double differential cross sections (DDCS) are presented. It is shown that for the $s_\Lambda-$wave hypernucleus production, the DCS is decreased with increasing nuclear mass, and the DCS for the $p_\Lambda-$wave hypernucleus production is normally higher than that for the $s_\Lambda-$wave hypernucleus production. As a reference, the DDCS with respect to the momenta of the outgoing proton and kaon is also demonstrated. Finally, the missing mass spectra of the inclusive reaction $p + A \to p + K^+ + X$ for ^6Li, ^{12}C and ^{16}O targets are presented, from which the masses of hypernuclei can accurately be extracted. Thus, we conclude that the missing mass spectrum method is an alternative to study hypernuclear physics. And the study of hypernuclear physics can be carried out in COSY and CSR by the $A(p, pK^+)_\Lambda B$ reaction due to the μb-order reaction cross sections.

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1 Introduction

There have been extensive advances in the field of hypernuclear physics. The conventional (K^-, π^-) strangeness exchange reactions and the associated production reactions of hypernuclei induced by energetic pion, photon and anti-proton beams have been studied experimentally and theoretically in last 30 years[1, 2, 3, 4, 5, 6, 7]. In the strangeness exchange reaction, the strange quark is brought in by the kaon beam and subsequently transferred to the target to form a hypernucleus. By choosing the incident momentum of the kaon, the momentum transfer can be very small for small π^- angles in this reaction, which favors the formation of low spin states of the hypernucleus. The use of energetic pion beams for the hypernucleus production was first proposed by Dover, Ludeking and Walker[4, 8] and further studied by Bando et al [9]. The associated (π^+, K^+) reaction, in which a pair of s and $\bar{s}$ quarks are produced, is endoergic, so the momentum transfer should be larger than the Fermi momentum for a pion momentum with which the two-body $\pi^+ n \to K^+ \Lambda$ cross section remains sizeable. The experimental studies of the (π^+, K^+) reactions at AGS and KEK clearly demonstrated the usefulness of this reaction for investigating the excitation of hypernuclei[6, 10, 11].

The hypernucleus can also be produced in p-A collisions. The possibility to produce hypernuclei in the (p, K^+) reaction was firstly mentioned by Yamazaki[12]. The possible associated-production reactions for hypernuclei in p-A collisions include the following types[13],

$$
\begin{array}{lrcl}
1) & p +_A Z & \to & {}^{A+1}_{\Lambda}Z + K^+, \\
2) & p +_A Z & \to & {}^{A}_{\Lambda}(Z-1) + p + K^+, \\
3) & p +_A Z & \to & K^+ + X.
\end{array}
\tag{1}
$$

Reaction 1) is a process with a two-body final state and is more suitable for high-resolution studies. Theoretical studies of the $A(p, K^+)_\Lambda B$ reaction were performed for some nuclei. The exclusive K^+ meson production in a proton-nucleus collision, leading to a two-body final state, is investigated in a fully covariant two-nucleon model based on an effective Lagrangian picture[14]. However, due to the large momentum transfer occurred in the reaction, the reaction cross sections are very small, which makes the experimental study very difficult. Reaction 3) is an inclusive process, where the outgoing K^+ is measured while the rest of the system in the final state X is unobserved. Some experiments for proton-induced-hypernucleus productions have been proposed in several laboratories[15, 16, 17, 18], but the experimental data and corresponding theoretical works are still sparse at this moment[19, 20, 21, 22, 23]. The production of Λ hypernuclei was studied in the reaction of the proton with the nucleus, and the lifetimes of the produced heavy hypernuclei were measured by the observation of delayed fission using the recoil shadow method[15, 16, 17] at COSY. At the same time, the experimental study of the production and decay of hypernuclei in p-A collisions was also carried out at Dubna[18].

Reaction 2) is a hypernuclear production process with a three-body final state in the p-A collision. This reaction was suggested for producing hypernuclei 35 years ago by Fetisov[24]. By choosing the momenta of the outgoing proton and K^+-meson, one can transfer a smaller momentum to the Λ hyperon, which is favorable for the formation of the Λ-hypernucleus. In the early work of Fetisov, the cross section of this reaction was estimated in the framework of the plane-wave impulse approximation and the cross section of the elementary process was taken from the early experimental data. It was found that the cross sections of these reactions are measurable with modern experimental technique.

In recent years, with the development of experimental technologies and methods, the proton beam with a substantially larger intensity is available. It provides an opportunity to re-examine the hypernuclear production in the three-body-final-state p-A collision. In this paper, we investigate this reaction in the framework of the distorted-wave-impulse-approximation (DWIA). The elementary process $p + p \to p + K^+ + \Lambda$ is calculated in a fully covariant two-nucleon model, where the effective Lagrangian, including contributions from the $N(1535)$, $N(1650)$, $N(1710)$ and $N(1720)$ excitations in the intermediate state, is employed.

In section 2, the theoretical model for the $A(p, pK^+)_\Lambda B$ reaction is briefly illustrated. Calculated results and analysis for the $A(p, pK^+)_\Lambda B$ reaction are presented in section 3. In section 4 the concluding remarks are given .

2 Theoretical model for $A(p, pK^+)_\Lambda B$ reactions

The production reaction of the Λ-hypernucleus in the p-A collision can proceed via a one-step process $pN \to K^+\Lambda N$ or a two-step cascade process in which a pion or a $\triangle$ is created in the intermediate stage, namely $pN \to \pi NN$ and $\pi N \to K^+\Lambda$ or $pN \to \triangle N$ and $\triangle N \to K^+\Lambda N$. However, it is commonly believed that the formation of the hypernucleus in the p-A collision is dominated by the one-step mechanism. Based on the one-step mechanism $pp \to pK^+\Lambda$, we study the $A(p, pK^+)_\Lambda B$ reaction in the framework of DWIA. In the reaction, we assume that the incoming proton interacts with one of the protons in the target and a N^* resonance is excited and followed by its decays into a K^+ meson and a Λ particle. Then, by the final-state interaction (FSI) between the Λ and the residual nucleus, the Λ-hypernucleus is formed. The relevant Feynman diagrams for the $A(p, pK^+)_\Lambda B$ reaction are depicted in Fig.1, where Figs.1(a) and (b) stand for the direct process in which the incident proton is excited into a N^* and for the exchange process which corresponds to the excitation of the target proton, respectively.

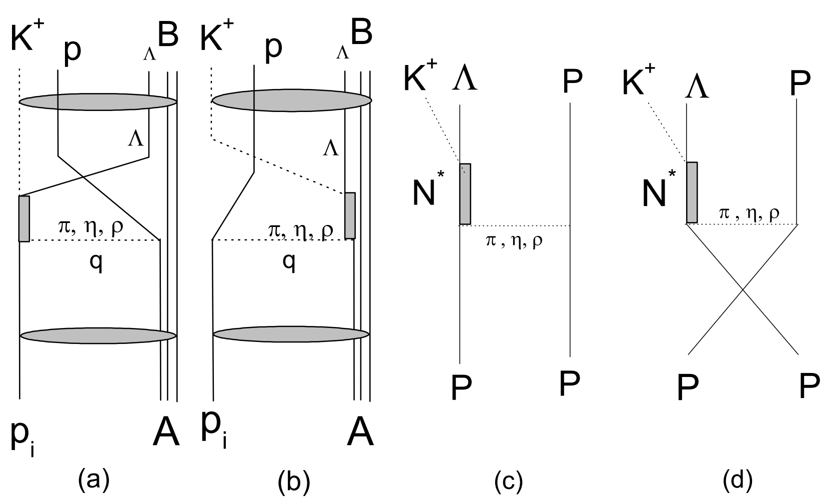

Figure 1: Ferynman diagrams for $pA \to pK^+{}_\Lambda B$ and $pp \to pK^+\Lambda$ reactions. The elliptic shaded areas represent the interactions in the incoming and outgoing channels. The rectangular shaded areas stands for nucleon resonances.

In the framework of DWIA the transition matrix for the $A(p, pK^+)_\Lambda B$ reaction can be written as

$$T_{fi}^{pA \to pK^+{}_\Lambda B} = \langle \Phi_f(\vec{r}_K, \vec{r}_0, \vec{r}_1, ..., \vec{r}_A)|\widehat{O}(\vec{r}_K, \vec{r}_0, \vec{r}_1, \vec{r}_2, ..., \vec{r}_A)|\Phi_i(\vec{r}_0, \vec{r}_1, ..., \vec{r}_A)\rangle \tag{2}$$

with

$$\Phi_i(\vec{r}_0,\vec{r}_1,...,\vec{r}_A)=\chi_p^{(+)}(\vec{P}_p,\vec{r}_0)\Psi_i(\vec{r}_1,...,\vec{r}_A), \tag{3}$$

$$\Phi_f(\vec{r}_0,\vec{r}_1,...,\vec{r}_A)=\chi_{p'}^{(-)}(\vec{P}_{p'},\vec{r}_0)\chi_K^{(-)}(\vec{P}_K,\vec{r}_K)\Psi_f(\vec{r}_1,...,\vec{r}_A), \tag{4}$$

where $\Phi_i(\vec{r}_0,\vec{r}_1,...,\vec{r}_A)$ and $\Phi_f(\vec{r}_K,\vec{r}_0,\vec{r}_1,...,\vec{r}_A)$ are the wave functions of the initial and final states, respectively. $\Psi_i(\vec{r}_1,...,\vec{r}_A)$ and $\Psi_f(\vec{r}_1,...,\vec{r}_A)$ denote the wave functions of the target nucleus and the hypernucleus, respectively. $\chi_p^{(+)}(\vec{P}_p,\vec{r}_0)$, $\chi_{p'}^{(-)}(\vec{P}_{p'},\vec{r}_0)$ and $\chi_K^{(-)}(\vec{P}_K,\vec{r}_K)$ represent the distorted wave functions of the incoming proton, the outgoing proton and the outgoing K^+ meson, respectively. $\vec{P}_p$ is the relative momentum of the incoming proton in the proton-nucleus c.m. system. $\vec{P}_{p'}$ and $\vec{P}_K$ are the momenta of the outgoing proton and K^+ meson, respectively. The transition operator $\widehat{O}$ can be written as,

$$\widehat{O}(\vec{r}_K,\vec{r}_0,\vec{r}_1,\vec{r}_2,...,\vec{r}_A)=\sum_{i=1}^{Z}v_{0i}(\vec{r}_0,\vec{r}_i,\vec{r}_K), \tag{5}$$

where v_{0i} is the interaction for the production of a K^+ meson in the collision of the incoming proton with a proton inside the nucleus, which is related to the t-matrix of the $pp\to pK^+\Lambda$ reaction in the impulse approximation. Z is the number of protons involved in the production process in the target nucleus.

Because the range of the nucleon-nucleon interaction is short compared with the extension of the nuclear wave function, one may assume that the incoming proton interacts with one of the nucleons in the target while the rest of the nucleons remain unchanged, which is a usual way to separate a single nucleon wave function from the many-body system. Then, the transition T-matrix for the hypernucleus production reaction reads as

$$T^{pA\to pK^+{}_\Lambda B}=Z\cdot F_{fi}(\vec{P}_{p'},\vec{P}_K;\vec{P}_p)\cdot t_{pp\to p\Lambda K^+}, \tag{6}$$

where $t_{pp\to p\Lambda K^+}$ stands for the t-matrix of the elementary K^+ production in p-p collisions and

$$F_{fi}(\vec{P}_{p'},\vec{P}_K;\vec{P}_p)=\int dV\chi_{p'}^{(-)*}(\vec{P}_{p'},\vec{r})\chi_K^{(-)*}(\vec{P}_K,\vec{r})\Phi^*_{J_HM_HT_HM_{T_H}}\Phi_{J_AM_AT_AM_{T_A}}\chi_p^{(+)}(\vec{P}_p,\vec{r}) \tag{7}$$

stands for the transition form factor that takes into account the effects from the distorted waves of incoming and outgoing particles and the wave functions of the target nucleus and hypernucleus. For a simple target nucleus where n nucleons stay in the j-subshell, we write the wave function of the nucleus as

$$\begin{aligned}\Phi_{J_A,M_A,T_A,M_{T_A}}(j^n(\alpha))=&\sum_{\alpha_0,J_0,T_0}<j^{n-1}(\alpha_0,J_0,T_0)(l)j\frac{1}{2}J_AT_A\,|\}j^n(\alpha)J_AM_AT_AM_{T_A}>\\ \times\sum_{M_0,M_{T_0}}&C^{J_AM_A}_{jM_A-M_0,\ J_0\ M_0}C^{T_AM_{T_A}}_{1/2M_{T_A}-M_{T_0},\ T_0M_{T_0}}\phi_{jM_A-M_0,\ 1/2M_{T_A}-M_{T_0}}\Phi_{J_0M_0,\ T_0M_{T_0}}(j^{n-1}(\alpha_0)),\end{aligned} \tag{8}$$

where$<j^{n-1}(\alpha_0,J_0,T_0)(l)jJ_AT_A\,|\}j^n(\alpha)J_AM_AT_AM_{T_A}>$ is the fractional parentage coefficient (f.p.c.). For the hypernucleus, we assume a weak coupling scheme, namely the Λ particle couples with a nuclear core to form a hypernuclear state, and express the wave function of the hypernucleus as

$$\Phi_{J_HM_HT_0M_{T_0}}(j^{n-1}(\alpha_0))=\sum_{M_0}C^{J_HM_H}_{j_\Lambda M_H-M_0,\ J_0M_0}\phi_{j_\Lambda M_H-M_0}\Phi_{J_0M_0,\ T_0M_{T_0}}(j^{n-1}(\alpha_0)). \tag{9}$$

Subsequently, the transition form factor becomes

$$
\begin{aligned}
& F_{fi}(\vec{P}_{p'},\vec{P}_K;\vec{P}_p) = < j^{n-1}(\alpha_0,J_0,T_0)(l)jJ_AT_A\,|\}j^n(\alpha)J_AM_AT_AM_{T_A}> \\
\times & \sum_{M_0,M_{T_0}} C^{J_AM_A}_{jM_A-M_0,\ J_0\ M_0} C^{T_AM_{T_A}}_{1/2M_{T_A}-M_{T_0},\ T_0M_{T_0}} C^{J_HM_H}_{j_\Lambda M_H-M_0,\ J_0M_0} \\
\times & \int d^3r \chi^{(-)*}_{p'}(\vec{P}_{p'},\vec{r})\chi^{(-)*}_K(\vec{P}_K,\vec{r})\phi^*_{j_\Lambda M_H-M_0}(\vec{r})\phi_{jM_A-M_0,\ 1/2M_{T_A}-M_{T_0}}(\vec{r})\chi^{(+)}_p(\vec{P}_p,\vec{r}). \quad (10)
\end{aligned}
$$

Moreover, as a very good approximation at higher energies[25, 26], we adopt the eikonal approximation to describe the distorted waves:

$$
\chi^{(+)}_p(\vec{P}_p,\vec{r}) = e^{i\vec{P}_p\cdot\vec{r}}\exp\left[-\frac{\sigma_p}{2}\int_{-\infty}^{z}\rho(b,z')dz'\right], \quad (11)
$$

$$
\chi^{(-)*}_{p'}(\vec{P}_{p'},\vec{r}) = e^{-i\vec{P}_{p'}\cdot\vec{r}}\exp\left[-\frac{\sigma_p}{2}\int_0^{\infty}\rho(b,z+l\frac{\vec{P}_{p'}}{P_{p'}})dl\right], \quad (12)
$$

$$
\chi^{(-)*}_K(\vec{P}_K,\vec{r}) = e^{-i\vec{P}_K\cdot\vec{r}}\exp\left[-\frac{\sigma_K}{2}\int_0^{\infty}\rho(b,z+l\frac{\vec{P}_K}{P_K})dl\right], \quad (13)
$$

where σ_p and σ_K are the $p-N$ and K^+-N total cross sections, respectively, and ρ the density distribution of the target nucleus.

The differential cross section of the $A(p,pK^+)_\Lambda B$ reaction with the initial nuclear state J_A and the final hypernucleus state J_H can be written as

$$
d\sigma^{((l)jJ_A,J_H)}(pA\to pK^+{}_\Lambda B) = \frac{(2\pi)^4}{4\sqrt{(P_p\cdot P_A)^2-(m_pm_A)^2}}\overline{\sum_{s_i}}\sum_{s_f}|T^{pA\to pK^+{}_\Lambda B}_{J_H,J_A}|^2 \quad d\phi_3, \quad (14)
$$

where $d\phi_3$ is the differential three-body phase space, S_i and S_f denote the spins of the initial and final states, respectively, P_p and P_A are the four momenta of the incoming proton and the target nucleus, respectively, and m_p and m_A are the masses of the proton and the nucleus, respectively. By integrating over relevant variables in Eq.(14), the measurable physical quantities, such as differential cross sections for the outgoing proton, kaon and hypernucleus, double differential cross sections, missing mass spectra of the reaction and etc., can be formulated. For example, the differential cross section and double differential cross section for the outgoing kaon can be written as

$$
\frac{d\sigma}{d\Omega_K} = \int\frac{E_pE_AE_KE_{p'}E_H}{(2\pi)^5|p_{pcm}|s}\overline{\sum_{s_i}}\sum_{s_f}|T^{pA\to pK^+{}_\Lambda B}_{J_H,J_A}|^2|\vec{P}^*_{p'}||\vec{P}_K|dm_{pH}d\Omega^*_{p'}, \quad (15)
$$

$$
\frac{d\sigma}{dP_Kd\Omega_K} = \int\frac{E_pE_A|\vec{P}_K|^2|\vec{P}_{p'}|^2}{(2\pi)^5|p_{pcm}|\sqrt{s}}\frac{1}{|\frac{dG}{d|\vec{P}_{p'}|}|}\overline{\sum_{s_i}}\sum_{s_f}|T^{pA\to pK^+{}_\Lambda B}_{J_H,J_A}|^2d\Omega_{p'}, \quad (16)
$$

where m_{pH} is the invariant mass of the outgoing proton and hypernucleus; p_{pcm} and $\sqrt{s}$ are the incident c.m. momentum and total energy; $\frac{dG}{d|\vec{P}_{p'}|}$ can be written as

$$
\frac{dG}{d|\vec{P}_{p'}|} = -\frac{|\vec{P}_{p'}|}{E_{p'}} - \frac{|\vec{P}_{p'}|+|\vec{P}_K|(\sin\theta_{p'}\cdot\sin\theta_K\cdot\cos(\varphi_{p'}-\varphi_K)+\cos\theta_{p'}\cdot\cos\theta_K)}{E_H}. \quad (17)
$$

Similar to the above derivations, we can obtain the single and double differential cross sections for the outgoing proton and hypernucleus, respectively. By integrating over all possible solid angles, we can further get double differential cross sections

$$\frac{d\sigma}{dP_p dP_K} = \int \frac{E_p E_A E_H |\vec{P}_{p'}||\vec{P}_K|}{(2\pi)^5 |P_{pcm}|\sqrt{s}} \overline{\sum_{s_i}} \sum_{s_f} |T^{pA \to pK^+ {}_\Lambda B}_{J_H, J_A}|^2 d\Omega_{p'} d\varphi_{p'K}, \tag{18}$$

where the $\varphi_{p'K}$ is the polar direction of $\vec{P}_K$ with respect to $\vec{P}_{p'}$. On the other hand, if we define invariant missing mass as $m_x^2 = (P_0 + P_A - P_p - P_K)^2$, the missing mass spectra for the four-body-final-state reaction of $p + A \to p + K^+ + \Lambda + B$ will be given as,

$$\frac{d\sigma}{dm_x} = \int \frac{\pi^2 E_p E_A E_{p'} E_K E_\Lambda E_{A'}}{16(2\pi)^8 |p_{pcm}| s^2 p_{p'K}} \overline{\sum_{s_i}} \sum_{s_f} |T^{pA \to pK^+ {}_\Lambda B}_{J_H, J_A}|^2 d\Omega_{p'} d\varphi_{p'K} d\varphi_\Lambda dm^2_{\Lambda A'} dm^2_{p'KA'} dm^2_{p'\Lambda A'}, \tag{19}$$

where $p_{p'K} = |\vec{P}_{p'} + \vec{P}_K|$; φ_Λ is the polar direction of $\vec{P}_\Lambda$ with respect to $\vec{P}_{p'} + \vec{P}_K$; $m_{\Lambda A'}$, $m_{p'KA'}$ and $m_{p'\Lambda A'}$ are respectively invariant masses and defined as $m^2_{\Lambda A'} = (P_\Lambda + P_{A'})^2$, $m^2_{p'KA'} = (P_{p'} + P_K + P_{A'})^2$ and $m^2_{p'\Lambda A'} = (P_{p'} + P_\Lambda + P_{A'})^2$.

Finally, by summing up all the quantum numbers for considered initial states of the proton in the target and possible states of the hypernucleus, the total reaction cross sections can be obtained by

$$\sigma^{tot}(p + A \to p + K^+ + {}_\Lambda B) = \sum_{(l)j, J_H} \sigma^{((l)jJ_A, J_H)}(p + A \to p + K^+ + {}_\Lambda B). \tag{20}$$

3 Numerical Results and Discussions

We first discuss the t-matrix of the elementary process $pp \to pK^+\Lambda$ which has been studied by many authors[27, 28, 29, 30] within the framework of the meson exchange theory. For example, Tsushima *et al.*[27] studied such a process by including the contributions from the $N^*(1650)$, $N^*(1710)$ and $N^*(1720)$ in the resonant model. The resultant total cross sections well agree with the experimental data at high energies. In order to better describe the data in the threshold region, an additional sub-threshold $N^*(1535)$ resonance was suggested by Liu *et al.*[28]. It was found that the $N^*(1535) - \Lambda - K^+$ coupling is quite large and the contribution from $N^*(1535)$ is very important for the cross sections of the $pp \to pK^+\Lambda$ reaction near the threshold. Based on these models, we re-calculate the amplitude of this elementary process by further considering the $p - \Lambda$ final-state interaction (FSI) [29]. In the calculation, the effective Lagrangians are chosen as those in Refs.[28, 27]. The resultant total cross sections for the $pp \to pK^+\Lambda$ reaction are plotted together with the experimental data [31] in Fig.2, where the solid and dashed curves denote the results with and without the contributions of the $N^*(1535)$ and the $p - \Lambda$ FSI, respectively. It is shown that by taking the contribution from the $N^*(1535)$ resonance and the $p - \Lambda$ FSI into account, the experimental data can be well described in both the high energy and low energy regions.

What we would like to know in this paper is that if concerned hypernuclei do exist, can we observe them in the $p + A \to p + K^+ + {}_\Lambda B$ reaction and accurately measure their mass spectrum for the study of the hyperon-nucleon (Y-N) interaction in future. For this purpose, we first employ a phenomenological Λ-nucleus potential to fit the masses of observed hypernuclei and

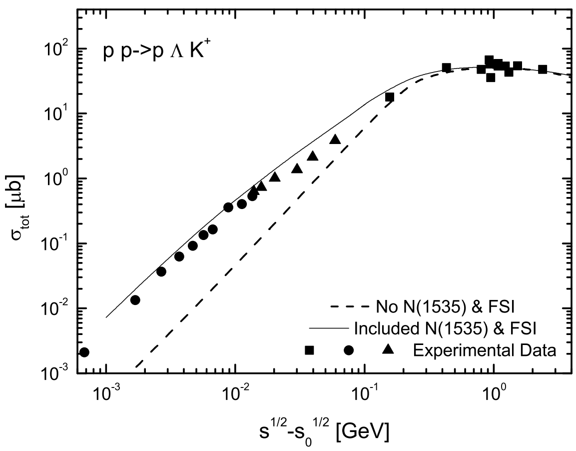

Figure 2: Total cross section of the $pp \to pK^+\Lambda$ reaction. $s^{1/2}$ and $s_0^{1/2} = m_N + m_\Lambda + m_K$ are the invariant collision energy and threshold energy, respectively. The experimental data are taken from Ref.[31]. The solid and dashed curves denote the results with and without the contributions of $N^*(1535)$ and $p\Lambda$ FSI, respectively.

obtain the bound state wave functions of the hypernuclei which will be studied in this paper. And then, with these wave functions, we calculate the measurable physical quantities to see if we can achieve our expectations.

As discussed in last section, we assume that the Λ particle interacts with a nucleus core to form a hypernuclear state . The bound-state wave function of the Λ is obtained by solving the Schrödinger equation with a phenomenological Λ-nucleus potential[32]

$$U_{\Lambda-A}(r) = -U_0 \frac{1}{1+e^{(r-R_0)/a}}, \tag{21}$$

where U_0=28MeV, $R_0 = (1.128 + 0.439A^{-2/3})$fm and a=0.54fm. The resultant binding energies of hypernuclei as well as the experimental data are tabulated in the second column in Table 1.

Table 1: The binding energies of Λ-hypernuclei. The corresponding experimental data are taken from[32, 33]. The l_Λ-state denoted the orbital angular momentum states of Λ-hyperon relative to core nucleus.

Hypernucleus (l_Λ-state)	Binding Energy(MeV)	
	solution of Schrödinger equation	Exp.
${}^6_\Lambda He$(s_Λ-state)	6.06	4.18
${}^{12}_\Lambda B$(s_Λ-state)	11.03	11.37
(p_Λ-state)	0.48	
${}^{16}_\Lambda N$(s_Λ-state)	12.97	(13)
(p_Λ-state)	2.51	

Then, we calculate the reaction cross sections of a proton bombarding on ${}^6Li$,${}^{12}C$ and ${}^{16}O$ targets. Same as that used in Refs.[34, 35], for the light target nucleus, we use the simple

single-particle wave function of harmonic oscillator, $\phi_{l_1 m_1}$, to describe target nucleons. For simplicity, we approximately take $< j^{n-1}(\alpha_0, J_0, T_0)(l) j J_A T_A \mid\} j^n(\alpha) J_A T_A >= 1$.

The nuclear distortion effects in the initial and final states are considered by using the eikonal approximation. We use the form of the harmonic-oscillator density(HO) for the nuclear charge distribution,

$$\rho(r) = \rho_0 \; [1 + \alpha(\frac{r}{a})^2] exp[-(\frac{r}{a})^2] \tag{22}$$

with the parameters determined from electron scatterings[36, 37]. Again for simplicity, we assume that the distributions for protons and neutrons in nuclei are the same. The total cross sections of two-body interactions, σ_p and σ_K, are taken from the experimental data at the corresponding relative momenta of the incident proton and outgoing kaon with respect to the target and residual nucleus, respectively[31].

It is worthy to mention that the differential reaction cross section is very sensitive to the form factor, and consequently to the momentum transfer, because the form factor is a function of the momentum transfer. When the momentum transfer increases, the form factor decreases rapidly. Thus, in order to reach a relatively large cross section for the hypernucleus production in the experiment, the energy of the incident proton should be limited in a range where the momentum transfer has the smallest value. In Fig.3, we show the minimum momentum transfer

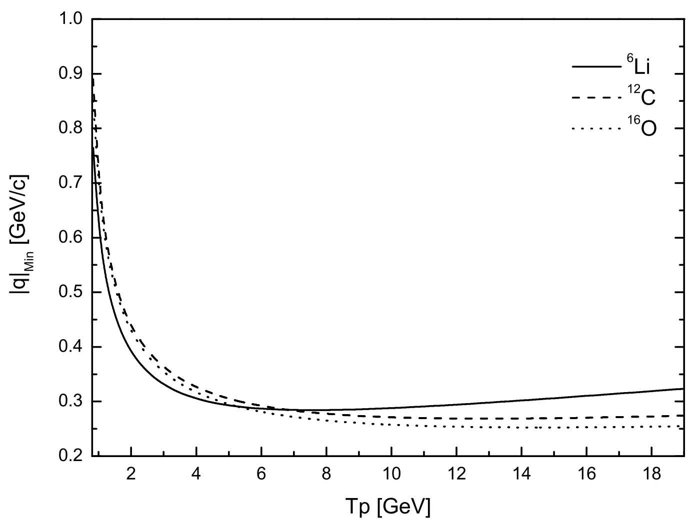

Figure 3: The minimum momentum transfers for reaction $^6Li(p, pK^+)^6_\Lambda He$, $^{12}C(p, pK^+)^{12}_\Lambda B$ and $^{16}O(p, pK^+)^{16}_\Lambda N$.

as a function of the kinetic energy of the incident proton for the reactions $^6Li(p, pK^+)^6_\Lambda He$, $^{12}C(p, pK^+)^{12}_\Lambda B$ and $^{16}O(p, pK^+)^{16}_\Lambda N$, respectively. We find that the minimum momentum transfer decreases with the increasing energy of the incident proton from the threshold of the reaction. When the kinetic energy of the proton is larger than 2 GeV, which is the available energy of the proton beam at COSY and the Cooling Storage Ring (CSR) at Lanzhou in China, the minimum momentum transfer becomes smaller than $300 - 400$MeV/c.

The calculated total cross sections(TCS) of the $^{12}C(p, pK^+)^{12}_\Lambda B$ reaction in the framework of DWIA are presented in Fig.4 and compared with the results in PWIA. In the calculation, the nuclear density in the HO form is employed for the evaluation of the distortions. We find that the distortion effects tend to reduce the cross section significantly by a factor of 3 to 10, and this effect will be more pronounced as the target nucleus becomes heavier.

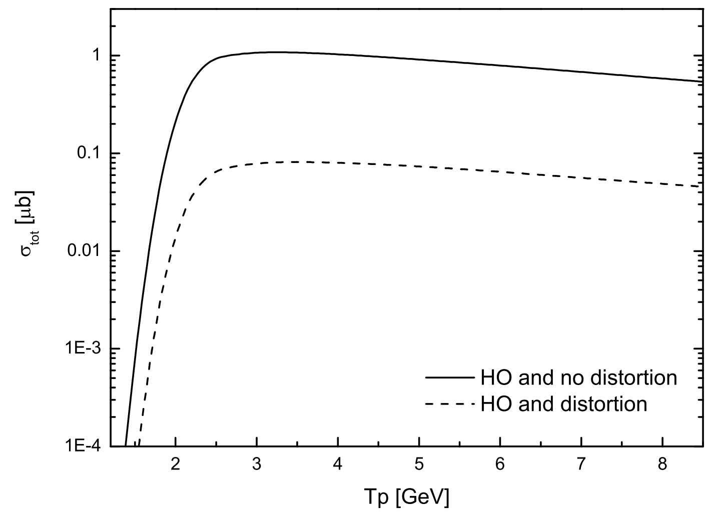

Figure 4: The total cross section of the ${}^{12}C(p, pK^{+})^{12}_{\Lambda}B$ reaction as a function of the incident proton energy. The solid and dashed curves represent the calculated results of TCS within the frameworks of PWIA and DWIA, respectively.

In fact, we are not able to detect the hypernucleus directly in experiment. The commonly used method to confirm the production of the hypernucleus is to detect the other concomitant particles. In the $A(p, pK^{+})_{\Lambda}B$ reaction, it is feasible to detect the outgoing proton and kaon.

The differential cross sections (DCS) in the c.m. system for the ${}^{6}Li$, ${}^{12}C$ and ${}^{16}O$ target nuclei in DWIA are presented in Fig.5, where diagrams (a), (b) and (c) denote the angular distributions for the outgoing proton, kaon and hypernucleus, respectively, and the kinetic energy of the incident proton, T_p, is 2.7GeV. From this figure, we find that the DCS for

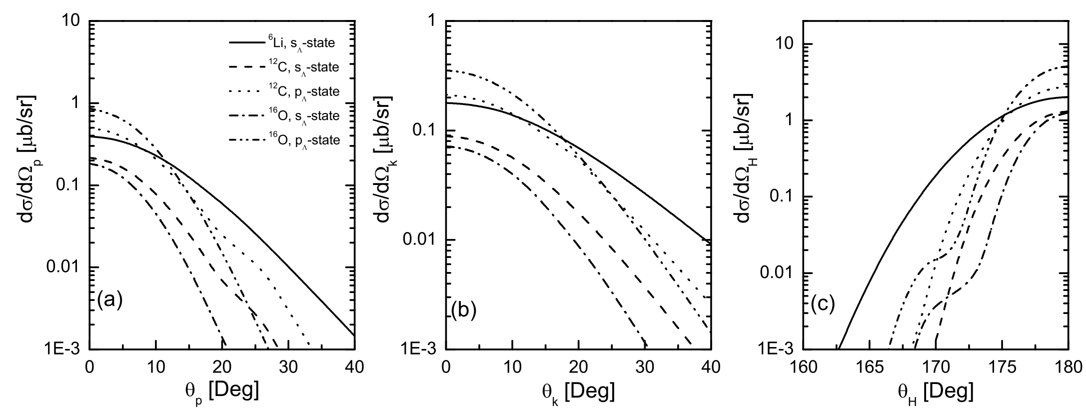

Figure 5: The differential cross sections in the c.m. system for the ${}^{6}Li$, ${}^{12}C$ and ${}^{16}O$ target nuclei in the framework of DWIA. The kinetic energy of the incident proton, T_p, is 2.7GeV.

the outgoing proton, as well as kaon, is larger at the forward angles, and for the resultant hypernucleus it is larger at the backward angles, because of the momentum conservation. For the same target, the DCS for the state with the p_{Λ}-shell Λ is generally larger than that with the s_{Λ}-shell Λ. This can be understood in the following way: Because the momentum transfer for the reaction $A(p, pK^{+})_{\Lambda}B$ is still relatively large at T_p=2.7GeV, the produced Λ would obtain a larger momentum, and consequently a higher orbital angular momentum[38, 39]. Then, the contribution from the conversions of the p-shell and s-shell nucleons to a p_{Λ}-shell Λ is larger

than that from the conversion of the p-shell and s-shell nucleons to a s_Λ-shell Λ. In the cases of ^6Li, ^{12}C and ^{16}O targets, the DCS of producing a s_Λ-state hypernucleus is normally smaller for the heavier target (larger A) than the one for the lighter target. This is because that the magnitude of the wave function product $\Phi^*_\Lambda\Phi_p$ becomes smaller with increasing nucleon number A. Thus, in the heavier target case, this factor, together with the distortion effect which is relatively larger in the heavier target case, would exceed the effect caused by the increased number of p-shell proton and make the DCS reduced. Moreover, for different targets, the DCS difference between the p_Λ-shell hypernucleus production case and the s_Λ-shell hypernucleus production case for the ^{16}O target is larger than that for the ^{12}C target. The reason for this phenomenon is that in the case of the s_Λ-shell hypernucleus production, as mentioned above, the DCS in the $^{16}O(p,\ pK^+)^{16}_\Lambda N$ reaction is suppressed in comparison the one in the $^{12}C(p,\ pK^+)^{12}_\Lambda B$ reaction. However, in the case of the p_Λ-shell hypernucleus production, the magnitude of $\Phi^*_\Lambda\Phi_p$ for the $^{16}O(p,\ pK^+)^{16}_\Lambda N$ reaction is almost the same as that for the $^{12}C(p,\ pK^+)^{12}_\Lambda B$ reaction, and the number of the p-shell proton in ^{16}O is larger than that in ^{12}C, so that the DCS for the p_Λ-shell hypernucleus production in the $^{16}O(p,\ pK^+)^{16}_\Lambda N$ reaction is larger than that in the $^{12}C(p,\ pK^+)^{12}_\Lambda B$ reaction.

In Fig.6, we show the dependance of the double differential cross sections (DDCS) on the outgoing proton momentum (a) and the outgoing kaon momentum (b), respectively. The results

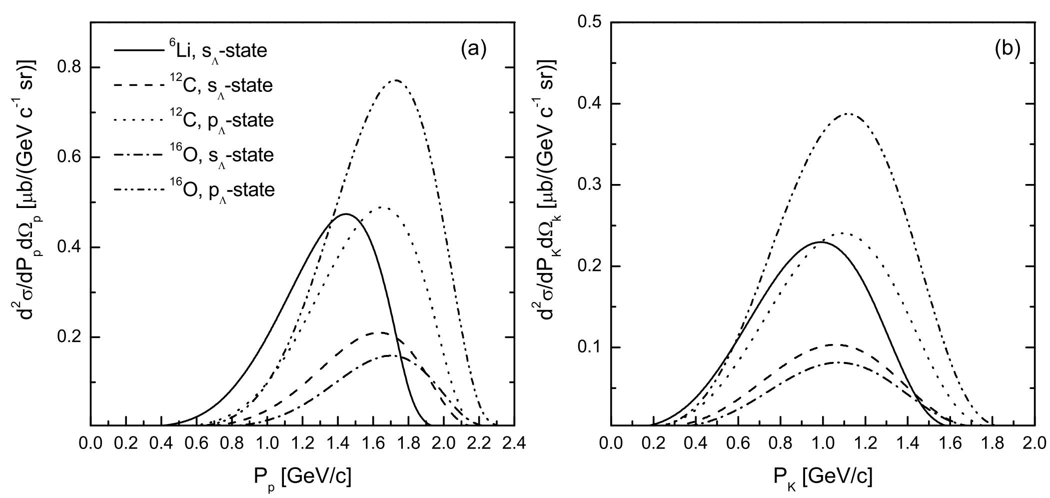

Figure 6: The double differential cross sections versus (a) the momentum of the outgoing proton at the angle 6° and (b) the momentum of the outgoing kaon at the angle 6°, in the C.M. system. The kinetic energy of incident proton is 2.7GeV.

show that for each hypernuclear production case, the DDCS has a Gaussian-like distribution with respect to the momentum of the outgoing proton or kaon. This means that there exist a momentum range of outgoing proton and a momentum range of kaon, where we can find the concomitant particles proton and kaon. Furthermore, when the target becomes heavier, the maximum DDCS point moves slightly to the higher proton momentum and kaon momentum regions. This enable produced Λ has smaller momentum so that it can be bound on the core nucleus which has less recoil due to its heavier mass. For a clear vision, we also present the DDCS with respect to the momentum and angle of outgoing kaon, respectively, for the ^{12}C target in Fig.7.

Since the final state of the reaction is a three-body state, one can confirm the production of the hypernucleus by detecting the outgoing proton and kaon simultaneously. We demonstrate

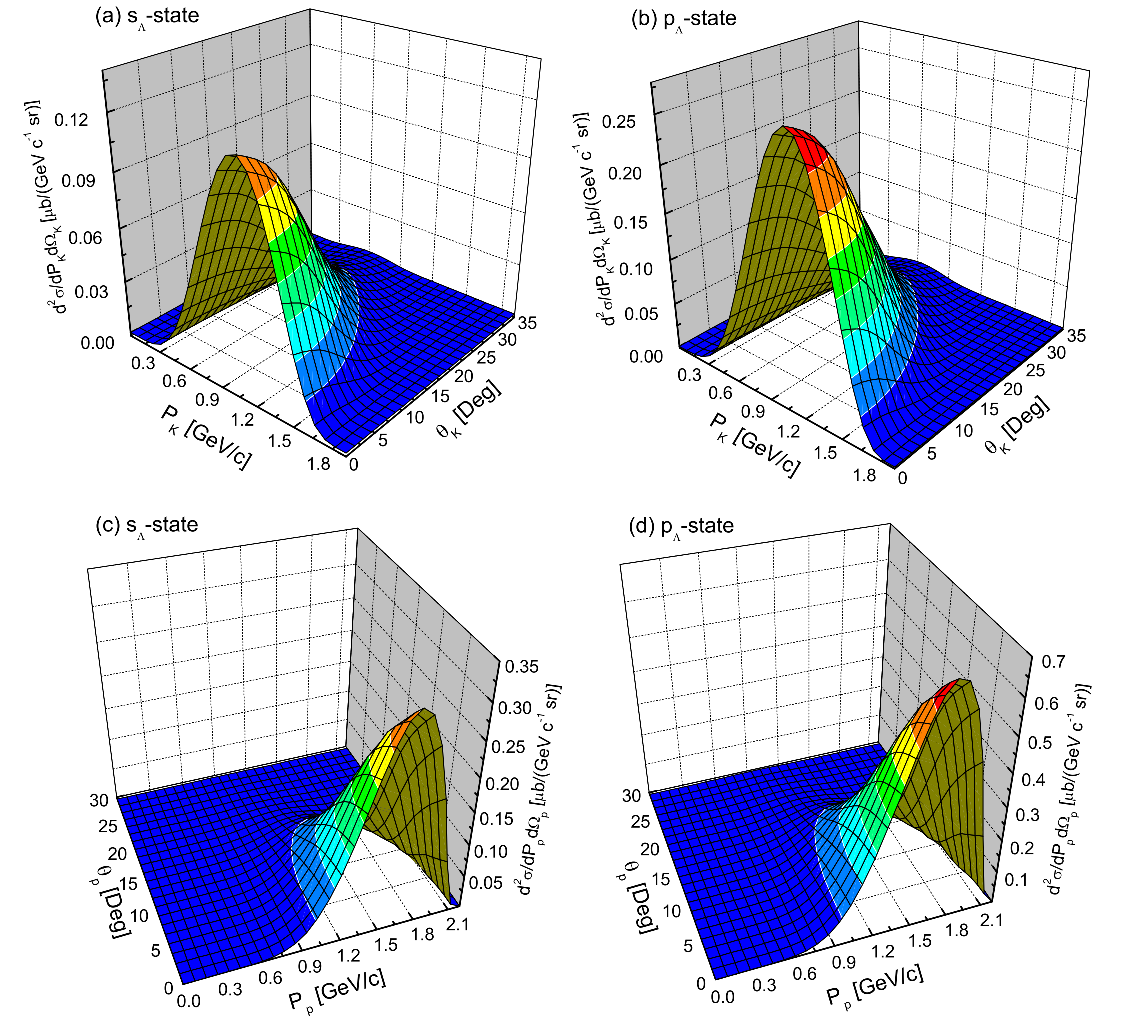

Figure 7: The 3D-plot for DDCS with respect to the momentum and angle of outgoing kaon ((a) and (b)), and proton ((c) and (d)), where (a) and (c) for s_Λ-shell hypernucleus and (b) and (d) for p_Λ-shell hypernucleus.

the DDCS distribution with respect to the momenta of the outgoing proton and kaon in Fig.8 From this figure, we find that DDCS is distributed in a special range due to the three-body phase space factor, which would be a reference for the experiment.

Furthermore, we study the mass and the formation signal of hypernucleus by analyzing the missing mass spectrum in the inclusive reaction $p + A \to p + K^+ + X$. The invariant missing mass can be defined as $m_x^2 = (P_0 + P_A - P - P_K)^2$. When the produced Λ can be bound on the core nucleus, namely the reaction $A(p, pK^+)_\Lambda B$ happens, the discrete peaks will appear in the missing mass spectrum. The invariant missing mass m_x corresponding to the peak position will associate with the mass of the produced hypernuclear state. We calculate the missing mass spectrum with the angles of outgoing proton and kaon to be limited in the range of $[0°, 12°]$ and plot the missing mass spectra for the ${}^6Li(p, pK^+)^6_\Lambda He$, ${}^{12}C(p, pK^+)^{12}_\Lambda B$ and ${}^{16}O(p, pK^+)^{16}_\Lambda N$ reactions in the left, middle and right diagrams of Fig.9, respectively. In the abscissa of this figure, $m_{A-1} = m_A - m_p - E_{B_p}$, where m_A, m_p and E_{B_p} are the mass of the target nucleus, the mass of the proton and the binding energy of the last proton in the target, respectively. One can see that each of the missing mass spectra contain two parts: discrete

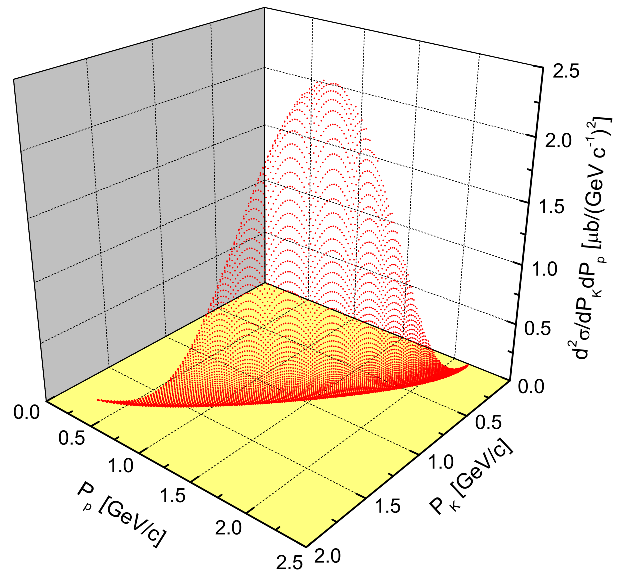

Figure 8: The c.m. double differential cross section distribution with respect to the momenta of the outgoing proton and kaon for the ^{12}C target nucleus in DWIA. The kinetic energy of incident proton is Tp=2.7GeV.

lines and a continual curve. The discrete lines express that the Λ particle is in a bound state with the residual nucleus, namely the final state is a three-body state. The continual curve is due to the contribution from a four-body final state. In Fig.9, the dotted curve denotes four-body differential cross sections in each case. In the left diagram, the single discrete line at the left-handed side of the four-body DCS curve indicates that only one s_Λ-wave hypernucleus state can be produced in the reaction with the ^6Li target. In the middle and right diagrams, the double discrete lines at the left-handed side of the four-body DCS curve indicate that one s_Λ-wave and one p_Λ-wave hypernuclear states can be produced in the reaction with the ^{12}C and the ^{16}O targets, respectively.

In the calculation of the missing mass spectra we use the calculated binding energies of the Λ tabulated in Table1 to obtain the masses of the hypernuclei. On the other hand, from the measurement one can extract the masses of the produced hypernuclei. Therefore, if the missing mass spectrum can accurately be measured, not only the masses of the produced hypernuclear states can accurately be determined, but also the hypernuclear production mechanism and the detailed hyperon-nucleon interaction, and consequently the hyperon-nucleus interaction, can authentically be studied.

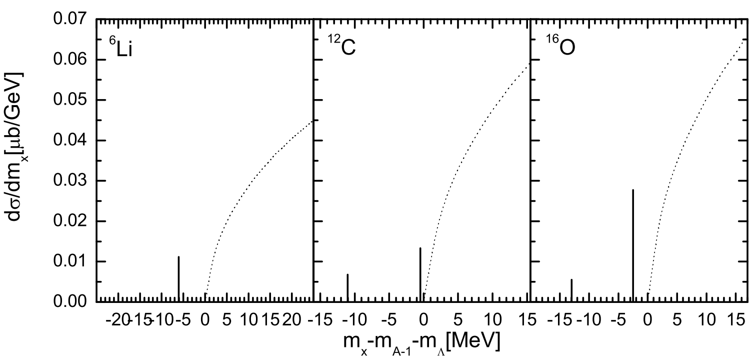

Figure 9: The missing mass spectra for the $^6Li(p, pK^+)^6_\Lambda He$, $^{12}C(p, pK^+)^{12}_\Lambda B$ and $^{16}O(p, pK^+)^{16}_\Lambda N$ reactions in DWIA at the kinetic energy of the incident proton Tp=2.7GeV. The angles of outgoing proton and kaon are limited in the range of $[0°, 12°]$.

4 Conclusions

We study the $A(p,pK^+)_\Lambda B$ reactions with several target nuclei in PWIA and DWIA. The elementary process $pp \to pK^+\Lambda$ is calculated in the resonance model in which the contributions from the exchanges of the π, η and ρ mesons, as well as those from the intermediate resonant states $N(1535)$, $N(1650)$, $N(1710)$ and $N(1720)$, are considered. Especially, by additionally including the contribution from the $N(1535)$ state and the $p\Lambda$ FSI, which provide a sizable effect, the total cross section data of the elementary process in the lower collision energy region can be much better reproduced.

In studying the $A(p,pK^+)_\Lambda B$ reactions in DWIA, the distortion effects for incident proton and outgoing kaon are obtained by the eikonal approximation. The Λ-nucleus wave function is calculated by solving the Schrödinger equation with a Woods-Saxon potential, with which the empirical binding energies of hypernuclei can be well reproduced.

The reaction cross sections for the ^6Li, ^{12}C and ^{16}O targets are calculated. It is shown that the nuclear effects are important and can reduce the cross section by factor of $3 \sim 10$. The partial wave differential cross sections are nuclear mass dependent. For the $s_\Lambda-$wave hypernucleus production, the DCS is decreased with increasing nuclear mass, and the DCS for the $p_\Lambda-$wave hypernucleus production is normally higher than that for the $s_\Lambda-$wave hypernucleus production. However, for the $p_\Lambda-$wave hypernucleus production, the DCS might not be decreased with increasing nuclear mass, due to complicated factors, such as the number of the $s_\Lambda-$shell protons in the target, the magnitude of the product $\Phi^*_\Lambda \Phi_p$, and etc.. The DDCS with respect to the momenta and the angles of the outgoing proton and kaon are presented, respectively. The three-dimensional plot for the DDCS with respect to the momenta of outgoing proton and kaon is also demonstrated as a experimental reference. It is noteworthy that the DDCS is distributed in a special range due to the three-body phase space factor.

Finally, we propose an alternative way to measure the masses of produced hypernuclei. That is the method of the missing mass spectrum. We present the missing mass spectrum of the inclusive process $p + A \to p + K^+ + X$. From this spectrum, we can accurately extract the masses of the hypernuclear states, if they can be formed. As a consequence, we can authentically study the hypernuclear production mechanism and the detailed hyperon-nucleon interaction, and consequently the hyperon-nucleus interaction.

As an conclusion, we believe that the reaction $A(p,pK^+)_\Lambda B$ is suitable for the hypernuclear physics study. These reactions can be carried out in COSY and CSR due to the μb-order reaction cross sections.

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References

[1] R. Bertini et al., Phys. Lett. B83(1979)306.

[2] H. C. Chiang and J. Hüfner, Phys. Lett. B84(1979)393.

[3] R. Bertini et al., Nucl. Phys. A360(1981)315.

[4] C. B. Dover, Phys. Rev. C22(1980)2073.

[5] T. Hasegawa et al., Phys. Rev. C53(1996)1210.

[6] T. Nagae, Eur. Phys. J. A18(2003)283.

[7] M. Agnelloa et al., Nucl. Phys. A752(2005)139.

[8] L.D. Ludeking, G.E. Walker, In Los Alamos 1980, Proceedings, Nuclear Structure With Intermediate-energy Probes, 286-291.

[9] H. Bando and T. Motoba, Prog. Theor. Phys. 76(1986)1321.

[10] H. Tamura(Tohoku U.) . Jan 2001. Prepared for Hirschegg '01: Structure of Hadrons: 29th International Workshop on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Austria, 14-20 Jan 2001. Published in Hirschegg 2001, Structure of hadrons 290-299.

[11] H. Tamura, Prog. Theor. Phys. Suppl. 156(2004)104.

[12] T. Yamazaki, Proc. KEK Internat. Workshop on Nuclear Physics in GeV Region, KEK Report 84-20, 3(1985).

[13] Carl B. Dover, "Nuclear Physics With Kaons", Scanned Version (KEK Library)198001280.

[14] R. Shyam *et al.*, Phys. Rev. C69(2004)065205.

[15] P. Kulessa *et al.*, Phys. Lett. B427 (1998)403.

[16] B. Kamys1 *et al.*, Eur.Phys. J A11(2001)1.

[17] P. Kulessa *et al.*, Acta Phys.Polon. B33(2002)603.

[18] S. A. Avramenko, Yu. A. Belikov, A. I. Golokhvastov *et al.*, Nucl.Phys.A585(1995)91c.

[19] Shoji Shinmura, Nucl. Phys. A450(1986)147c.

[20] B. V. Krippa, Z.Phys. A351(1995)411.

[21] V. I. Komarov, A. V. Lado *et al.*, J Phys. G21(1995)L69; arXiv: nucl-th/9804050.

[22] Anders Gardestig, arXiv:nucl-th/9606059 v1 28 Jun 1996.

[23] V. N. Fetisov, Nucl. Phys. A639(1998)177c.

[24] V. N. Fetisov *et al.*, Phys. Lett. B38 (1972)129.

[25] B. K. Jain *et al.*, Phys. Rev. C47(1993)1701.

[26] S. Hirenzaki, E. Oset *et al.*, Phys. Lett. B378(1996)29.

[27] K. Tsushima *et al.*, Phys. Rev. C59(1999)369; arXiv:nucl-th/9801063.

[28] B. C. Liu, B. S. Zou, Phys. Rev. Lett. 96(2006)042002.

[29] A. Sibirtsev *et al.*, arXiv:nucl-th/0004022.

[30] R. Shyam, Phys. Rev. C60(1999)055213.

[31] Landolt-Börnstein, *New Series*, edited by H. Schopper,I/12(1988); J. M. Hauptmann *et al.*, Nucl. Phys. B125(1977)29; K. J. Anderson *et al.*, Phys. Rev. D11(1975)473; J. Balewski *et al.*, Phys. Lett. B 420(1998)211; S. Sewerin *et al.*, Phys. Rev. Lett. 83 (1999) 682; P. Kowina *et al.*, Eur.Phys.J. A22(2004)293.

[32] D. J. Millener *et al.*, Phys. Rev. C38(1988)2700.

[33] L. Majling, Nucl. Phys. A585(1995)211c; Scanned Version(KEKlibary) 199305173.

[34] C. J. Horowitz and B. D. Serot, Nucl. Phys. A368(1981)503.

[35] Tan Yu-hong *et al.*, Chin. Phys. Lett. 18(2001)1030.

[36] C. W. De Jager, H. De Vries *et al.*, Atomic Data and Nuclear Data Tables 14(1974)479.

[37] H. De Vries *et al.*, Atomic Data and Nuclear Data Tables 36(1987)495.

[38] Jing Hantao *et al.*,"Production of Λ-hypernuclei in $A(p, K^+)_\Lambda B$ Reaction", will be published.

[39] C. B. Dover, L. Ludeking and G. E. Walker, Phys. Rev. C22(1980)2073.